Week 2: Algebra Basics, Formulae and Functions

Solutions

1. Simplify, if possible, (a) $\dfrac{abc}{3ac}$ (b) $\dfrac{3ab}{a+b}$

   Solution: (a) $\frac{abc}{3ac} = \frac{b}{3}$ (b) $\frac{3ab}{a+b}$ cannot simplify further

2. Simplify, if possible, $\dfrac{x^2+2x-15}{2x^2-5x-3}$

   Solution: $\dfrac{x^2+2x-15}{2x^2-5x-3} = \dfrac{(x+5)(x-3)}{(2x+1)(x-3)} = \dfrac{x+5}{2x+1}$

3. Transpose $v = \sqrt{x+2y}$, (a) for x, (b) for y.

   Solution: (a) $x = v^2 - 2y$ (b) $y = (v^2 - x)/2$

4. The surface area of a sphere is given by the formula $SA = 4\pi r^2$. If the sphere has a surface area of 20 cm^2, what is the radius of the sphere?

   Solution: $r = 1.26$ cm

5. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.

   (a) Calculate the volume of a cone with radius 4cm and height 5cm.

   (b) Rearrange the formula to make h the subject.

   (c) Rearrange to make r the subject.

   (d) What height is a cone whose radius is 2.4 cm and whose volume is 37 cm^3.

   Solution: (a) $83.8 cm^3$ (b) $h = \frac{3V}{\pi r^2}$ (c) $r = \sqrt{\frac{3V}{\pi h}}$ (d) 6.13cm

6. Given two functions $g(t) = 3t + 2$ and $h(t) = t + 3$ obtain an expression for (a) the composition $g(h(t))$, and (b) the composition $h(g(t))$, and (c) $g(g(t))$.

   Solution: (a) $g(h(t)) = 3t + 11$ (b) $h(g(t)) = 3t + 5$ (c) $g(g(t)) = 9t + 8$

7. State the vertical intercept and the gradient of each of the following lines:

   Solution:

   (a) $y = 3x + 3$ Intercept $= 3$, grad $= 3$

   (b) $y = 2x - 3$ Intercept $= -3$, grad $= 2$

   (c) $y = 4$ Intercept $= 4$, grad $= 0$

   (d) $y = 1 - x$ Intercept $= 1$, grad $= -1$

   (e) $y = -5x$ Intercept $= 0$, grad $= -5$

8. Sketch the lines from question 1. Which has the steepest gradient? Where do lines a. and c. intersect?

   Solution: Steepest gradient is line e (gradient = -5), steepest positive gradient is line a (gradient = 3).

   Lines a. and c. intersect where 3x+3=2x-3 i.e. at (-6, -15).
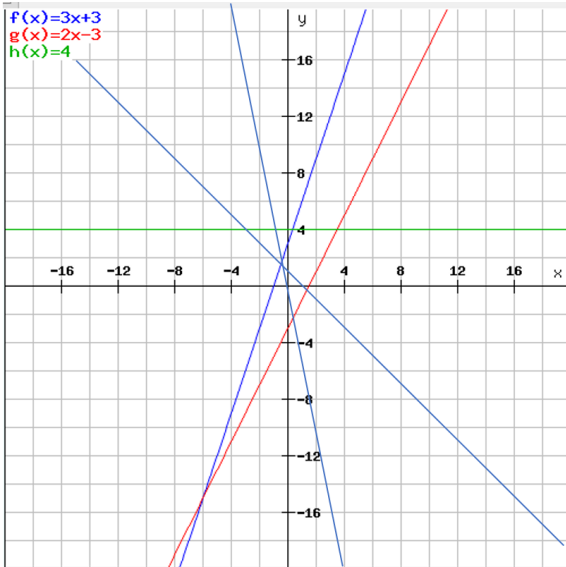
9. Which of these are straight lines?

   (a) $2x + 3y = 4$ Yes

   (b) $y = 3x^2 + 5$ No

   (c) $4xy + 2 = 5$ No

   (d) $x = 3$ Yes

   (e) $x + y = 1.2$ Yes

   (f) $x^2 - y^2 = 2$ No

10. What is the gradient of the straight line through (1,2) and (3,5)?

    Solution: $\frac{3}{2} = 1.5$

11. What is the equation of the straight line in question 10?

    Solution: $y = \frac{3}{2}x + \frac{1}{2}$

12. What is the distance between the points in question 10?

    Solution: $d = \sqrt{2^2 + 3^2} = \sqrt{13} = 3.61$