business maths foundations

6.7

Confidence Intervals

1. Introduction

Suppose we took a sample of 50 students and found their mean height. A different sample of 50 students would probably have a different mean height.

- A sample is only an approximation for a population.

We can use the sample mean to calculate a range within which the population mean is likely to fall.

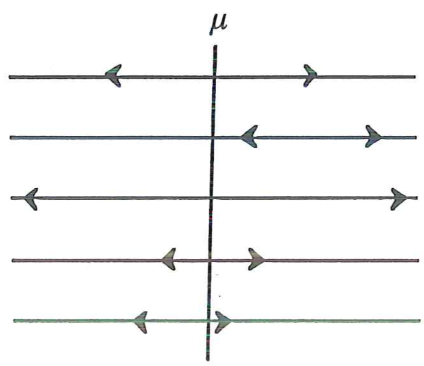
- This range is generally taken to be 95% i.e. a **95% Confidence Interval.**

There is only one true value of the population mean. The Confidence Interval is the likely range of that value. It is **not** the variability of the true value.

The probability this interval contains the population mean is 0.95.

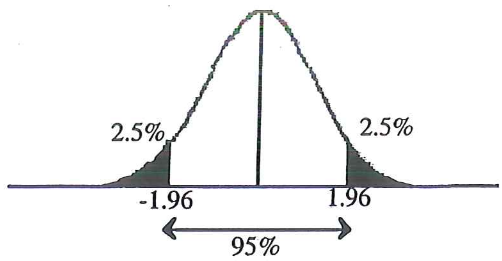
We can be 95% confident that this range contains the population mean.

Suppose we took 5 different samples and calculated the Confidence Interval for the population mean, we would expect these Confidence Intervals to contain the population mean in "95% of cases".



2. Confidence Intervals

From Normal tables, we can look up the z values we require. For 95% Confidence Interval we find the confidence limits are ± 1.96.

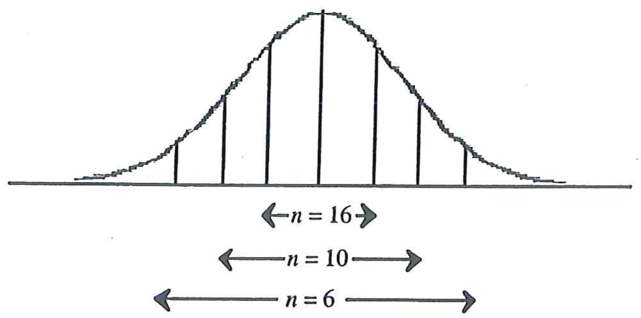


$P(-1.96 \leq \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \leq 1.96) = 0.95$

Rearranging it can be shown:

$P\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$

Confidence Interval	Confidence Limits
90%	± 1.645
95%	± 1.96
99%	± 2.576



As the sample size increases, the Confidence Interval decreases.

Worked example 1

If x is normally distributed with mean 10.39 and standard deviation 2, and $n = 12$, find a 95% confidence interval.

$x \sim N(10.39, 2^2), \quad n = 12$

95% confidence limits are:

$$10.39 - 1.96 \times \frac{2}{\sqrt{12}} \quad \text{and} \quad 10.39 + 1.96 \times \frac{2}{\sqrt{12}}$$

$$= 9.26 \qquad\qquad = 11.52$$

95% Confidence Interval $= (9.26, 11.52)$.

Worked example 2

If x is normally distributed with mean 5.3 and standard deviation 1.5, and $n = 100$, find a 99% confidence interval.

$x \sim N(5.3, 1.5^2), \quad n = 100$

99% confidence limits are:

$$5.3 - 2.576 \times \frac{1.5}{\sqrt{100}} \quad \text{and} \quad 5.3 + 2.576 \times \frac{1.5}{\sqrt{100}}$$

$$= 4.91 \qquad\qquad = 5.69$$

99% Confidence Interval $= (4.91, 5.69)$.

Exercises

For the following find (i) 90% Confidence Interval (ii) 95% Confidence Interval (iii) 99% Confidence Interval.

a) $X \sim N(140, 2^2), \quad n = 60$

b) $X \sim N(76, 12^2), \quad n = 100$

c) $X \sim N(748, 3.6^2), \quad n = 150$

Answers

a) (139.6, 140.4), (139.5, 140.4), (139.3, 140.7)

b) (74.0, 78.0), (73.6, 78.4), (72.9, 79.1)

c) (747.5, 748.5), (747.4, 748.6), (747.2, 748.8)